SUPERFRACTALTHING MATHS

K.I. MARTIN

The standard Mandelbrot set equation can be written as:
$X_{n+1} = X_n^2 + X_0$

Where the complex number X_0 is in the Mandelbrot set if X_n stays finite as n tends to infinity. Traditionally, when creating mandelbrot set images, points in the Mandelbrot set are coloured black, and points outside have a colour generated from the minimum value of n required to give the complex number X_n a magnitude of more than 2.

Consider another point given by Y_0
$Y_{n+1} = Y_n^2 + Y_0$

The difference between these two points at a given iteration is given by Δ_n such that

$\Delta_n = Y_n - X_n$

Then

$\Delta_{n+1} = Y_{n+1} - X_{n+1}$
$\Delta_{n+1} = (Y_n^2 + Y_0) - (X_n^2 + X_0)$
$\Delta_{n+1} = ((X_n + \Delta_n)^2 + X_0 + \Delta_0) - (X_n^2 + X_0)$
$\Delta_{n+1} = (X_n^2 + 2X_n\Delta_n + \Delta_n^2 + X_0 + \Delta_0) - (X_n^2 + X_0)$

$\Delta_{n+1} = 2X_n\Delta_n + \Delta_n^2 + \Delta_0$ **(1)**

Equation (1) is important, as all the numbers are 'small', allowing it to be calculated with hardware floating point numbers. So if we know all the values X_n, we can use this equation to calculate Y_n without having to use arbitrary precision calculations.

Let $\delta = \Delta_0$

$\Delta_1 = 2X_0\delta + \delta^2 + \delta = (2X_0 + 1)\delta + \delta^2$
$\Delta_2 = (4X_1X_0 - 2X_1 - 1)\delta + ((x_0 - 1)^2 + 2X_1)\delta^2 + (4X_0 - 2)\delta^3 + o(\delta^4)$

Let $\Delta_n = A_n\delta + B_n\delta^2 + C_n\delta^3 + o(\delta^4)$ **(2)**

Then

$A_{n+1} = 2X_nA_n + 1$ **(3)**
$B_{n+1} = 2X_nB_n + A_n^2$ **(4)**
$C_{n+1} = 2X_nC_n + 2A_nB_n$ **(5)**

Now we can apply equations (3), (4) and (5) iteratively to calculate the coefficents for equation (2). Equation (2) can then be used to calculate the value for the n^{th} iteration for all the points around X_0. The approximation should be good as long as the δ^3 term has a magnitude significantly smaller then the δ^2 term.